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Reflection and transmission of weakly inhomogeneous anisotropic and bianisotropic layers calculated by perturbation method

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Abstract
With the use of perturbation theory the zero- and higher-order approximations are obtained for characteristic matrices, reflection and transmission operators, scalar reflectance and transmittance of monochromatic electromagnetic waves impinging on a transparent weakly inhomogeneous stratified bianisotropic medium. The proposed procedure of perturbation series formation enables finding the corrections to the reflection and transmission factors in closed form for specified coordinate dependencies for the material tensors of the bianisotropic layers. The general expressions obtained are applied to calculate the first-order corrections for normal incident light on (i) a twisted uniaxial slab, (ii) an isotropic medium with arbitrary inhomogeneity profile and (iii) a one-dimensional photonic crystal with arbitrary number of layers. Exact and approximate solutions of the wave equations are numerically compared.

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1. Introduction

Bianisotropic media constitute the most general class of linear media, in which the electric induction $\boldsymbol{D}$ and the magnetic field strength $\boldsymbol{H}$ depend on the electric field strength $\boldsymbol{E}$ and the magnetic induction $\boldsymbol{B}$ [1–3]. Constitutive equations for such media are linear and tensorial and can be presented in different equivalent forms [3, 4]. In a number of cases the constitutive equations are used, where vectors $\boldsymbol{D}$ and $\boldsymbol{B}$ are expressed in terms of $\boldsymbol{E}$ and $\boldsymbol{H}$. Simpler media belonging to the class of bianisotropic ones (bi-isotropic reciprocal and nonreciprocal, anisotropic and isotropic media) are characterized by fewer material constants involved in the constitutive equations. Various kinds of the constitutive equations for bianisotropic and gyrotropic media along with optical properties and effects of frequency and spatial dispersion in these media are discussed in numerous works (see, for example, [1–17, 43, 46]).

In recent years, the attention of many researches is focused on artificial non-conventional materials with unusual optical properties such as left-handed media [18–20], photonic crystals [21–23], metamaterials based on carbon nanotubes [24] and others. From the viewpoint of classical electrodynamics the properties of such materials can be explained both by structure of constitutive equations (for example, simultaneously negative dielectric permittivity and magnetic permeability for left-handed media) and by inhomogeneity and non-stationarity of the materials (i.e. spatial and temporal dependence of the material parameters). The simplest, but nonetheless practically important instance of inhomogeneous media are one-dimensional inhomogeneous or stratified media. In general, inhomogeneity in one dimension can be mathematically described by piecewise continuous tensorial functions of one spatial coordinate for the material parameters (particular cases are continuous and piecewise constant functions). It is known that Maxwell's equations for the monochromatic electromagnetic field in a stratified anisotropic medium can be reduced to the system of the first-order ordinary differential equations for the tangential components of the field [25, 26] (see also [27, 28, 37, 38, 43]). For arbitrary coordinate dependence of the material parameters it is impossible to find closed-form solutions. There is a variety of exact and approximate techniques for analysis of electromagnetic wave propagation in stratified structures including the bianisotropic ones. In particular, those are Brillouin zone theory [29], approximation of geometrical optics [30–32], operator methods [33–46] based on Fedorov's works on $SO(3)$ covariant crystal optics and crystal acoustics [3, 47–49], Green function techniques [50–54], vector circuit theory [55], wave splitting in the time-domain [52, 56, 57], transformation equation [58] and vector Fresnel equation [59] techniques; eigenwave technique for description of x-ray reflection and diffraction [60] and others. A number of new methods is used to reveal interdependence between symmetry of multilayered media including fractal ones and their spectral periodicity [61–64]. In papers [33–35], the three-dimensional operators of spatial evolution for electromagnetic field were introduced for the first time which are in essence optical analogues of quantum-mechanical temporal evolution operators for Schrödinger's equation. Later on the evolution operators were used for solving a number of problems in optics of stratified media [37, 38, 42–44, 46]. In [35, 37, 41, 43], the covariant impedance tensor formalism was developed for calculation of light reflection and refraction by inhomogeneous structures; in [36, 43] an operator-based multiple reflection method was proposed for the analysis of light propagation in stratified anisotropic and bianisotropic media, generalizing the existing analogous scalar methods [65]. In the papers [39], operator generalization was extended to the basic concepts of geometrical optics of stratified media.

One of the widespread methods of modern physics is the Green function technique [66] and perturbation theory associated with it. We do not dwell here on numerous references concerning these problems. Note that the perturbation methods are productively used both in classical and quantum physics. The use of these methods in quantum physics is based on a possibility to divide a Hamiltonian into two parts. One of them is the unperturbed part H_0 and the other is the addition λH_1 with a small parameter λ. It is assumed that the spectrum of the unperturbed Hamiltonian H_0 can be calculated in some way. As a result it turns out to compose a formal perturbation series in terms of powers of λ.

In many practically important optical applications anisotropic or bianisotropic media are weakly inhomogeneous, i.e. the medium characteristics change only slightly at the distances of the order of wavelength. It is evident that in this case the system matrix of the wave differential equations of the first order [26, 43] can be divided into the basic coordinate-independent part corresponding to some effective homogeneous medium and the coordinate-dependent small addition. The perturbation methods enable to find the Green function for the system of equations under consideration.

The purpose of this paper is to develop the perturbation series for the spatial evolution operators (characteristic matrices), the reflection and transmission operators as well as the scalar reflectance and transmittance of monochromatic light obliquely incident on a transparent weakly inhomogeneous bianisotropic layer surrounded by a homogeneous isotropic medium. We use the operator methods to calculate the reflection and transmission of inhomogeneous media [37, 38, 43]. In section 2, general expressions for the system matrix elements of wave equations are given for the case of oblique light incidence on the bianisotropic layer characterized by material tensors $\varepsilon(\omega)$, $\mu(\omega)$, $\alpha(\omega)$ and $\beta(\omega)$. The reflection and transmission operators are expressed in terms of the evolution operator (propagator) of the layer and the surface impedance tensors of the incident, reflected and transmitted waves. For arbitrary light polarization including partial polarization the reflection and transmission factors are presented in covariant form. These factors are quadratic forms of the corresponding reflection and transmission operators. In section 3, we compose a formal perturbation series for the propagator of the layer, then we consider eigenvectors of the unperturbed propagator and find the matrix elements of the propagator for any order of perturbation theory. These elements correspond to contribution of a pure eigenwave that leaves the layer if an entering eigenwave is specified. With the use of the series for the propagator we construct the corresponding series for the reflection and transmission factors. In the zeroth order these factors coincide with reflectance and transmittance of the effective homogeneous bianisotropic layer. The obtained relations are used in section 4 to calculate the first-order correction to the reflectance and transmittance of an uniformly twisted uniaxial slab for normal light incidence. Inhomogeneity of the slab is due to spatial rotation of the principal axes of the dielectric permittivity tensor but the principal values of this tensor remain coordinate independent. Continuous axial twisting can be caused by the material structure and/or external action. In particular, helical twisted structure is typical for cholesteric liquid crystals [67–69, 27] studied in many works (see, for example, [70–73]). In the zero-order approximation we substitute the slab for an effective isotropic medium with the refraction index coincident with that of ordinary waves in the slab. The formal perturbation series is also used to calculate the first-order corrections to the reflectance and transmittance of a weakly inhomogeneous isotropic medium with arbitrary profile of dielectric permittivity $\varepsilon(z)$ for normal light incidence (section 5). This case is of great importance for different applications of atmosphere radiophysics, optoelectronics and modern nanoscience [27, 65]. General expressions for these corrections involve Fourier-transform components of $\varepsilon(z)$. Then they are used to derive reflection and transmission of a one-dimensional photonic crystal with an arbitrary number of alternating layers with different refractive indices n_1 and n_2. In section 6, we compare numerically the zero- and first-order approximate reflectance with the exact one. Dependence of the approximate and exact reflection factors on several parameters of a twisted crystal is studied. These parameters are the slab thickness, difference of the principal values of the dielectric permittivity tensor (small perturbation parameter), refractive index of the cladding isotropic medium, orientation of light polarization and spatial period of twisting. For weakly inhomogeneous photonic crystals we compare the approximate and exact reflection factors depending on the electromagnetic wave frequency close to photonic band gaps. Finally, we discuss the basic results of the paper in section 7.

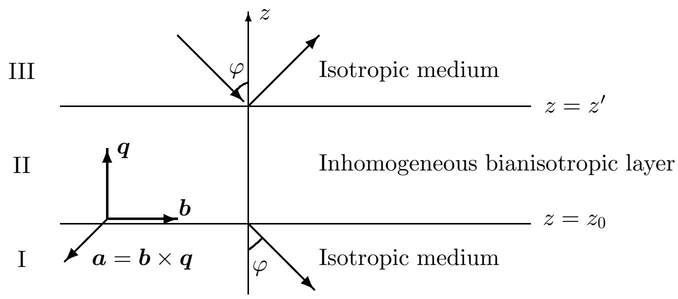

Figure 1. Reflection and refraction of an electromagnetic wave by the bianisotropic layer (φ is the angle of incidence).

2. Fresnel's reflection and transmission operators

Consider oblique incidence of a plane electromagnetic wave with frequency ω on a bianisotropic layer II surrounded by a homogeneous isotropic medium with dielectric permittivity ε' and magnetic permeability μ' (domains I and III, figure 1). Let the z-axis of Cartesian coordinates be directed perpendicular to the layer, planes $z = z_0$ and $z = z'$ are interfaces, the layer is inhomogeneous in the z-direction. Let us introduce the right-handed triple of unit vectors $\boldsymbol{b}$, $\boldsymbol{q}$ and $\boldsymbol{a} = \boldsymbol{b} \times \boldsymbol{q}$, where $\boldsymbol{q}$ is the normal vector to the layer directed along the z-axis, the vectors $\boldsymbol{b}$ and $\boldsymbol{q}$ determining the plane of incidence. Spatial dependence of the electromagnetic field vectors is described by equations

$$\{\boldsymbol{E}(\boldsymbol{r}), \boldsymbol{D}(\boldsymbol{r}), \boldsymbol{H}(\boldsymbol{r}), \boldsymbol{B}(\boldsymbol{r})\} = \{\boldsymbol{E}(z), \boldsymbol{D}(z), \boldsymbol{H}(z), \boldsymbol{B}(z)\} \exp(\mathrm{i}\kappa \boldsymbol{b}\boldsymbol{r}),$$

where κ is the projection of the wave vector onto the $\boldsymbol{b}$-direction. In particular, for normal wave incidence $\kappa = 0$.

Constitutive equations for the wave field in the bianisotropic layer have the form [3, 4]

$$\boldsymbol{D} = \varepsilon(\omega)\boldsymbol{E} + \alpha(\omega)\boldsymbol{H}, \qquad \boldsymbol{B} = \beta(\omega)\boldsymbol{E} + \mu(\omega)\boldsymbol{H}, \tag{1}$$

where the three-dimensional tensors ε and μ characterize dielectric and magnetic properties of the layer at frequency ω, while the tensors α and β describe magnetoelectric properties. These tensors are functions of z-coordinate ($z_0 < z < z'$).

From Maxwell's equations supplemented by relations (1) it follows that the tangential components of the electromagnetic field within the layer are subjected to the following matrix system of the first-order differential equations:

$$\frac{\mathrm{d}U(z)}{\mathrm{d}z} = \mathrm{i}k_0 N(z) U(z), \qquad N(z) = \begin{pmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{pmatrix}, \qquad U(z) = \begin{pmatrix} \boldsymbol{H}_\tau \\ \boldsymbol{q} \times \boldsymbol{E} \end{pmatrix}, \tag{2}$$

where $k_0 = \omega/c$; $\boldsymbol{H}_\tau = I\boldsymbol{H} = -\boldsymbol{q} \times (\boldsymbol{q} \times \boldsymbol{H})$ is the tangential component of $\boldsymbol{H}$; $I = -\boldsymbol{q}^{\times 2} = 1 - \boldsymbol{q} \otimes \boldsymbol{q}$ is the projective operator; 1 is the unit tensor of three-dimensional space; sign $\otimes$ marks the direct (tensor, dyadic) product of vectors; $\boldsymbol{q}^\times$ is the tensor dual to the vector $\boldsymbol{q}$ ($\boldsymbol{q}^\times_{im} = e_{ijm} q_j$ [3, 47], e_{ijm} is the completely antisymmetric Levi–Civita pseudotensor).

Tensorial elements of matrix $N(z)$ are expressed in terms of ε, μ, α and β in an intricate manner (see, for example, [43])

$$
\begin{aligned}
N_{11} &= \boldsymbol{q}^{\times}\alpha I - \frac{1}{\Delta}(\varepsilon_q \boldsymbol{b}_1 \otimes \boldsymbol{q}\mu I - \mu_q \boldsymbol{q}^{\times}\varepsilon\boldsymbol{q} \otimes \boldsymbol{a}_2 - \alpha_q \boldsymbol{q}^{\times}\varepsilon\boldsymbol{q} \otimes \boldsymbol{q}\mu I + \beta_q \boldsymbol{b}_1 \otimes \boldsymbol{a}_2),\\
N_{12} &= -\boldsymbol{q}^{\times}\varepsilon\boldsymbol{q}^{\times} - \frac{1}{\Delta}(\varepsilon_q \boldsymbol{b}_1 \otimes \boldsymbol{b}_2 - \mu_q \boldsymbol{q}^{\times}\varepsilon\boldsymbol{q} \otimes \boldsymbol{q}\varepsilon\boldsymbol{q}^{\times} - \alpha_q \boldsymbol{q}^{\times}\varepsilon\boldsymbol{q} \otimes \boldsymbol{b}_2 + \beta_q \boldsymbol{b}_1 \otimes \boldsymbol{q}\varepsilon\boldsymbol{q}^{\times}),\\
N_{21} &= I\mu I - \frac{1}{\Delta}(\varepsilon_q I\mu\boldsymbol{q} \otimes \boldsymbol{q}\mu I + \mu_q \boldsymbol{a}_1 \otimes \boldsymbol{a}_2 + \alpha_q \boldsymbol{a}_1 \otimes \boldsymbol{q}\mu I + \beta_q I\mu\boldsymbol{q} \otimes \boldsymbol{a}_2),\\
N_{22} &= -I\beta\boldsymbol{q}^{\times} - \frac{1}{\Delta}(\varepsilon_q I\mu\boldsymbol{q} \otimes \boldsymbol{b}_2 + \mu_q \boldsymbol{a}_1 \otimes \boldsymbol{q}\varepsilon\boldsymbol{q}^{\times} + \alpha_q \boldsymbol{a}_1 \otimes \boldsymbol{b}_2 + \beta_q I\mu\boldsymbol{q} \otimes \boldsymbol{q}\varepsilon\boldsymbol{q}^{\times}),
\end{aligned}
\tag{3}
$$

where the following vectors:

$$
\boldsymbol{a}_1 = \frac{\kappa}{k_0}\boldsymbol{a} - I\beta\boldsymbol{q}, \qquad \boldsymbol{a}_2 = \frac{\kappa}{k_0}\boldsymbol{a} - \boldsymbol{q}\alpha I, \qquad \boldsymbol{b}_1 = \frac{\kappa}{k_0}\boldsymbol{b} + \boldsymbol{q}^{\times}\alpha\boldsymbol{q}, \qquad \boldsymbol{b}_2 = \frac{\kappa}{k_0}\boldsymbol{b} - \boldsymbol{q}\beta\boldsymbol{q}^{\times}
$$

and scalars $\varepsilon_q = \boldsymbol{q}\varepsilon\boldsymbol{q}$, $\mu_q = \boldsymbol{q}\mu\boldsymbol{q}$, $\alpha_q = \boldsymbol{q}\alpha\boldsymbol{q}$, $\beta_q = \boldsymbol{q}\beta\boldsymbol{q}$, $\Delta = \varepsilon_q\mu_q - \alpha_q\beta_q$ are introduced. If a solution of the system of equations (2) is found, then one can restore the complete vectors $\boldsymbol{E}$ and $\boldsymbol{H}$ from the known components $\boldsymbol{H}_\tau$ and $\boldsymbol{q} \times \boldsymbol{E}$ as

$$
\begin{pmatrix} \boldsymbol{H}(z) \\ \boldsymbol{E}(z) \end{pmatrix} = V(z)U(z), \qquad V(z) = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}, \tag{4}
$$

where the elements of the restoration matrix V have the form

$$
\begin{aligned}
V_{11} &= I - \frac{1}{\Delta}\boldsymbol{q} \otimes (\varepsilon_q \boldsymbol{q}\mu I + \beta_q \boldsymbol{a}_2), & V_{12} &= -\frac{1}{\Delta}\boldsymbol{q} \otimes (\varepsilon_q \boldsymbol{b}_2 + \beta_q \boldsymbol{q}\varepsilon\boldsymbol{q}^{\times}),\\
V_{21} &= \frac{1}{\Delta}\boldsymbol{q} \otimes (\mu_q \boldsymbol{a}_2 + \alpha_q \boldsymbol{q}\mu I), & V_{22} &= -\boldsymbol{q}^{\times} + \frac{1}{\Delta}\boldsymbol{q} \otimes (\mu_q \boldsymbol{q}\varepsilon\boldsymbol{q}^{\times} + \alpha_q \boldsymbol{b}_2).
\end{aligned}
\tag{5}
$$

The elements of matrix N are planar tensors, i.e. $N_{ij}\boldsymbol{q} = \boldsymbol{q}N_{ij} = 0$, $N_{ij}I = IN_{ij} = N_{ij}$, $i, j = 1, 2$.

Equations (2) and (3) are covariant (i.e. they are invariant with respect to transformations of rotation group $SO(3)$), and have the same form regardless of the choice of basis in three-dimensional space. In particular, if the x-axis, y-axis and z-axis are directed along vectors $\boldsymbol{b}$, $-\boldsymbol{a}$ and $\boldsymbol{q}$, respectively, and $\mu = 1$, $\alpha = \beta = 0$ then equations (2) can be reduced to the known Berreman's equations [26] for components H_x, H_y, $-E_y$, E_x.

Note that in [45] the matrix system (2) was represented in an equivalent form. The elements of matrix N were expressed in terms of tensorial bilinear forms of vectors $\boldsymbol{b}$ and $\boldsymbol{q}$. The obtained system of equations has been used for general analysis of the surface wave propagation in linear bianisotropic stratified media and derivation of the dispersion relations for surface polaritons.

If the field $U(z_0)$ at the interface $z = z_0$ is known, then the field in any other point with coordinate z ($z > z_0$) can be found with the use of an evolution operator (propagator, characteristic matrix) $P(z, z_0)$ according to the formula

$$
U(z) = P(z, z_0)U(z_0). \tag{6}
$$

It is evident that $P(z, z_0)$ satisfies the equation analogous to (2)

$$
\frac{\mathrm{d}P(z, z_0)}{\mathrm{d}z} = \mathrm{i}k_0 N(z)P(z, z_0), \tag{7}
$$

and in addition $P(z_0, z_0) = E$, where

$$
E = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}. \tag{8}
$$

Using the propagator of the bianisotropic layer $P \equiv P(z', z_0)$ and the surface impedance tensor γ which relates the vectors $\boldsymbol{H}_\tau$ and $\boldsymbol{q} \times \boldsymbol{E}$ according to the formula

$$
\boldsymbol{q} \times \boldsymbol{E} = \gamma\boldsymbol{H}_\tau, \tag{9}
$$

it is not difficult to calculate Fresnel's reflection and transmission operators r and d [37, 43] of the layer. These operators are planar ($rI = Ir = r, dI = Id = d$) and they enable finding the tangential components of the magnetic field strength for the reflected $\boldsymbol{H}_\tau^{\mathrm{r}}$ and transmitted $\boldsymbol{H}_\tau^{\mathrm{d}}$ wave in terms of the tangential component of the incident wave field $\boldsymbol{H}_\tau^{\mathrm{i}}$:

$$\boldsymbol{H}_\tau^{\mathrm{r}} = r\boldsymbol{H}_\tau^{\mathrm{i}}, \qquad \boldsymbol{H}_\tau^{\mathrm{d}} = d\boldsymbol{H}_\tau^{\mathrm{i}}. \tag{10}$$

We have $PU^{\mathrm{d}} = U^{\mathrm{i}} + U^{\mathrm{r}}$ or in expanded form, considering (9) and (10)

$$P\begin{pmatrix} I \\ \gamma^{\mathrm{d}} \end{pmatrix} d\boldsymbol{H}_\tau^{\mathrm{i}} = \begin{pmatrix} I \\ \gamma^{\mathrm{i}} \end{pmatrix} \boldsymbol{H}_\tau^{\mathrm{i}} + \begin{pmatrix} I \\ \gamma^{\mathrm{r}} \end{pmatrix} r\boldsymbol{H}_\tau^{\mathrm{i}}, \tag{11}$$

where γ^{i}, γ^{r} and γ^{d} are surface impedance tensors of the incident, reflected and transmitted wave, respectively, in the surrounding isotropic medium [37, 43]. Since the z-projections of the incident and transmitted wave vector are the same, and the projection of the reflected wave vector is opposite in sign, then $\gamma^{\mathrm{r}} = -\gamma^{\mathrm{i}} = -\gamma^{\mathrm{d}} \equiv \gamma$. Multiplying equation (11) from the left first by matrix $(\gamma \quad -I)$ and then by matrix $(\gamma \quad I)$ and omitting $\boldsymbol{H}_\tau^{\mathrm{i}}$, we obtain

$$r = (I \quad \gamma^{-})P\begin{pmatrix} I \\ -\gamma \end{pmatrix}\left[(I \quad -\gamma^{-})P\begin{pmatrix} I \\ -\gamma \end{pmatrix}\right]^{-}, \qquad d = 2\left[(I \quad -\gamma^{-})P\begin{pmatrix} I \\ -\gamma \end{pmatrix}\right]^{-}, \tag{12}$$

where the sign $^{-}$ denotes pseudoinversion of a planar tensor (i.e. $\gamma^{-}\gamma = \gamma\gamma^{-} = I$).

From (2) and (9) it follows that the surface impedance tensor γ obeys the tensorial Riccati equation

$$\frac{\mathrm{d}\gamma}{\mathrm{d}z} + \mathrm{i}k_0(\gamma N_{11} + \gamma N_{12}\gamma - N_{21} - N_{22}\gamma) = 0. \tag{13}$$

The tensor γ involved in equations (12) is related to the isotropic medium that surrounds the layer and is characterized by scalar parameters ε', μ' ($\alpha' = \beta' = 0$). Therefore from (3) we have $N_{11} = N_{22} = 0$, $N_{12} = \varepsilon' I - \kappa^2\boldsymbol{b}\otimes\boldsymbol{b}/(\mu' k_0^2)$, $N_{21} = \mu' I - \kappa^2\boldsymbol{a}\otimes\boldsymbol{a}/(\varepsilon' k_0^2)$ for this medium. Moreover, the medium is homogeneous so γ does not depend on z and in view of (13) satisfies algebraic equation $\gamma N_{12}\gamma = N_{21}$. In consideration of $\boldsymbol{b}\otimes\boldsymbol{b} + \boldsymbol{a}\otimes\boldsymbol{a} = I$ we have

$$\gamma = N_{12}^{-}\sqrt{N_{12}N_{21}} = \frac{\mu'\boldsymbol{b}\otimes\boldsymbol{b}}{\sqrt{\varepsilon'\mu' - \kappa^2/k_0^2}} + \frac{1}{\varepsilon'}\sqrt{\varepsilon'\mu' - \frac{\kappa^2}{k_0^2}}\boldsymbol{a}\otimes\boldsymbol{a} \tag{14}$$

(see also [43]). Taking into account that $\kappa/k_0 = \sin\varphi\sqrt{\varepsilon'\mu'}$, where φ is the angle of incidence of the wave we arrive at another representation of the tensor γ

$$\gamma = \sqrt{\frac{\mu'}{\varepsilon'}}\left(\frac{\boldsymbol{b}\otimes\boldsymbol{b}}{\cos\varphi} + \cos\varphi\,\boldsymbol{a}\otimes\boldsymbol{a}\right). \tag{15}$$

Thus, the equations (12) and (14) (or (15)) completely determine the reflection and transmission operators r and d provided that the layer propagator P is evaluated beforehand.

As a rule, what one measures experimentally is the scalar reflection and transmission factors R and D equal to the ratio of the reflected and transmitted wave intensity, respectively, to the incident wave intensity. They are quadratic functions of the operators r and d. It is convenient to calculate these factors using the beam tensor (coherence tensor) [3], which is determined as follows:

$$\Phi = \sum_s \boldsymbol{H}_s \otimes \boldsymbol{H}_s^*,$$

where the subscript s enumerates the separate incoherent constituents of the wave with amplitudes $\boldsymbol{H}_s$. The wave intensity is found as a trace of the coherence tensor $\mathcal{I} = \operatorname{tr}\Phi$.

Let an incident wave consist of incoherent waves with amplitudes $\boldsymbol{H}_s^{\mathrm{i}}$. Then the refracted wave will consist of the waves with amplitudes $\boldsymbol{H}_s^{\mathrm{r}} = v^{\mathrm{r}} r I \boldsymbol{H}_s^{\mathrm{i}} = v^{\mathrm{r}} r \boldsymbol{H}_s^{\mathrm{i}}$, where v^{r} is the three-dimensional restoration operator of the vector $\boldsymbol{H}_s^{\mathrm{r}}$ from its tangential component $\boldsymbol{H}_{\tau s}^{\mathrm{r}}$. The coherence tensor of the refracted wave is

$$\Phi^{\mathrm{r}} = \sum_s v^{\mathrm{r}} r \boldsymbol{H}_s^{\mathrm{i}} \otimes \boldsymbol{H}_s^{\mathrm{i}*} r^{+} v^{\mathrm{r}+} = v^{\mathrm{r}} r \Phi^{\mathrm{i}} r^{+} v^{\mathrm{r}+},$$

where Φ^{i} is the coherence tensor of the incident wave, the sign $^{+}$ denoting Hermitian conjugation. Without loss of generality we put the incident wave intensity to unity. Then the reflection factor is

$$R = \operatorname{tr}(v^{\mathrm{r}+} v^{\mathrm{r}} r \Phi^{\mathrm{i}} r^{+}). \tag{16}$$

Starting from the equations (4) and (9), we ascertain that $v^{\mathrm{r}} = V_{11} + V_{12}\gamma^{\mathrm{r}} = V_{11} + V_{12}\gamma$, where the tensor γ has the form (15), and according to (5) for the isotropic medium $V_{11} = I, V_{12} = -\kappa \boldsymbol{q} \otimes \boldsymbol{b}/(\mu' k_0) = -\sin\varphi\sqrt{\varepsilon'/\mu'}\boldsymbol{q} \otimes \boldsymbol{b}$. Then $v^{\mathrm{r}} = I - \tan\varphi \boldsymbol{q} \otimes \boldsymbol{b}$, and equation (16) takes the form

$$R = \operatorname{tr}(w r \Phi^{\mathrm{i}} r^{+}), \tag{17}$$

where

$$w = v^{\mathrm{r}+} v^{\mathrm{r}} = I + \tan^2\varphi \boldsymbol{b} \otimes \boldsymbol{b} = \frac{\boldsymbol{b} \otimes \boldsymbol{b}}{\cos^2\varphi} + \boldsymbol{a} \otimes \boldsymbol{a}. \tag{18}$$

Analogously considering the transmitted wave ($\gamma^{\mathrm{d}} = -\gamma$), we find the transmittance of the layer

$$D = \operatorname{tr}(w d \Phi^{\mathrm{i}} d^{+}), \tag{19}$$

where the tensor w is determined by the same formula (18).

General relations (17) and (19) can be applied for light that is linear polarized in some direction $\boldsymbol{h}$ ($|\boldsymbol{h}|^2 = 1, \boldsymbol{h}\boldsymbol{n} = 0, \boldsymbol{n} = \sin\varphi \boldsymbol{b} - \cos\varphi \boldsymbol{q}$ is a phase normal), as well as for nonpolarized and partially polarized light. For the first case $\Phi^{\mathrm{i}} = \boldsymbol{h} \otimes \boldsymbol{h}^*$. For nonpolarized light $\Phi^{\mathrm{i}} = -\frac{1}{2}\boldsymbol{n}^{\times 2}$. Finally, if light is partially polarized then Φ^{i} is represented by sum of 'polarized' and 'nonpolarized' parts [3]: $\Phi^{\mathrm{i}} = p\boldsymbol{h} \otimes \boldsymbol{h}^* - \frac{1}{2}(1-p)\boldsymbol{n}^{\times 2}$, where p is the polarization degree ($0 \leqslant p \leqslant 1$).

Generally it is impossible to find analytically exact solution of system (2) even if one of the tensors ε, μ, α or β involved in (1) depends arbitrarily on z. So the approximate solution techniques including perturbation method are of great importance.

3. Calculation of the layer propagator by perturbation method

Perturbation methods are widely used in modern physics and are expounded in numerous monographs and papers. Here, we briefly outline the basic steps of constructing the perturbation series for the matrix $P(z', z_0)$, taking into account specific features of light transmission through a bianisotropic layer.

Let the bianisotropic layer be weakly inhomogeneous, i.e. components of tensors ε, μ, α and β slightly change in the interval from z_0 to z':

$$\begin{aligned} &\max_{z_0<z<z'} \varepsilon_{ij} - \min_{z_0<z<z'} \varepsilon_{ij} \ll 1, \qquad \max_{z_0<z<z'} \mu_{ij} - \min_{z_0<z<z'} \mu_{ij} \ll 1, \\ &\max_{z_0<z<z'} \alpha_{ij} - \min_{z_0<z<z'} \alpha_{ij} \ll 1, \qquad \max_{z_0<z<z'} \beta_{ij} - \min_{z_0<z<z'} \beta_{ij} \ll 1, \end{aligned} \tag{20}$$

$i, j = 1, 2, 3$. It enables a decomposition of the matrix $N(z)$ (2) into a constant part, which does not depend on z, and a part $\mathcal{N}(z)$ with matrix elements on the order of magnitude of the differences in (20):

$$N(z) = N_0 + \mathcal{N}(z). \tag{21}$$

As a matter of fact, the choice of the matrix N_0 is ambiguous. Additional constrains for N_0 will be discussed later.

The perturbation series for $P(z', z_0)$ has the form

$$P(z', z_0) = P^{(0)}(z' - z_0) + P^{(1)}(z', z_0) + \cdots + P^{(n)}(z', z_0) + \cdots, \tag{22}$$

at $z' > z_0$ the nth term of the series determined by the expression ($n \geqslant 1$)

$$\begin{aligned} P^{(n)}(z', z_0) = (\mathrm{i}k_0)^n \int_{z_0}^{z'} \mathrm{d}z_n \int_{z_0}^{z_n} \mathrm{d}z_{n-1} \cdots \int_{z_0}^{z_3} \mathrm{d}z_2 \int_{z_0}^{z_2} \mathrm{d}z_1 P^{(0)}(z' - z_n)\mathcal{N}(z_n) \\ \times P^{(0)}(z_n - z_{n-1}) \cdots \mathcal{N}(z_2) P^{(0)}(z_2 - z_1)\mathcal{N}(z_1) P^{(0)}(z_1 - z_0). \end{aligned} \tag{23}$$

Equations (22) and (23) contain the zero-order approximation of the propagator

$$P^{(0)}(z) = \mathrm{e}^{\mathrm{i}k_0 z N_0}, \tag{24}$$

which is a solution of equation (7) for $N(z) = N_0$.

All upper limits of integration in (23) can be changed to z' if the retarded Green's function is used

$$P_{\mathrm{c}}(z) = \theta(z) P^{(0)}(z) = \theta(z)\, \mathrm{e}^{\mathrm{i}k_0 z N_0}, \tag{25}$$

where $\theta(z)$ is the Heaviside unit step function ($\theta(z) = 1$ for $z \geqslant 0$ and $\theta(z) = 0$ for $z < 0$). Then

$$\begin{aligned} P^{(n)}(z', z_0) = (\mathrm{i}k_0)^n \int_{z_0}^{z'} \mathrm{d}z_n \int_{z_0}^{z'} \mathrm{d}z_{n-1} \cdots \int_{z_0}^{z'} \mathrm{d}z_2 \int_{z_0}^{z'} \mathrm{d}z_1 P^{(0)}(z' - z_n)\mathcal{N}(z_n) \\ \times P_{\mathrm{c}}(z_n - z_{n-1}) \cdots \mathcal{N}(z_2) P_{\mathrm{c}}(z_2 - z_1)\mathcal{N}(z_1) P^{(0)}(z_1 - z_0). \end{aligned} \tag{26}$$

The Green's function satisfies the equation

$$\frac{\mathrm{d}P_{\mathrm{c}}(z - z_0)}{\mathrm{d}z} = \mathrm{i}k_0 N_0 P_{\mathrm{c}}(z - z_0) + \delta(z - z_0) E,$$

where $\delta(z)$ is the Dirac delta function.

In choosing the matrix N_0 we assume that it refers to the effective homogeneous bianisotropic layer with the material tensors $\varepsilon_0, \mu_0, \alpha_0, \beta_0$ which is only slightly different from $\varepsilon(z), \mu(z), \alpha(z), \beta(z)$. Consider the eigenvectors U_m of N_0 belonging to the four-dimensional complex amplitude space $\mathbb{C}^4 = \mathbb{C}^2 \oplus \mathbb{C}^2$ (a direct sum of the two-dimensional spaces orthogonal to $\boldsymbol{q}$, see explicit form of U in (2)):

$$N_0 U_m = n_m U_m, \qquad m = 1, \ldots, 4. \tag{27}$$

The unit operator of $\mathbb{C}^4$ is represented by the matrix E (8). In this case, the eigenvalues n_m are wave vector projections of the eigenwaves onto the $\boldsymbol{q}$-direction in the units of k_0. For normal wave incidence they simply coincide with the refractive indices of the eigenwaves.

For our further use it is essential to establish the conditions that lead to the quantities n_m being real. A necessary (but not sufficient) condition of that is the following set of constraints on the tensors $\varepsilon_0, \mu_0, \alpha_0, \beta_0$:

$$\varepsilon_0^+ = \varepsilon_0, \qquad \mu_0^+ = \mu_0, \qquad \beta_0^+ = \alpha_0. \tag{28}$$

It means that absorption in the homogeneous layer is absent [3]. If conditions (28) are fulfilled then it follows from (3) that $(N_0)^+_{12} = (N_0)_{12}$, $(N_0)^+_{21} = (N_0)_{21}$, $(N_0)^+_{11} = (N_0)_{22}$ or

$$TN_0 = N_0^+ T, \qquad T = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}. \tag{29}$$

From (27) and (29) we ascertain that $U_m^+ T$ is the left eigenvector of the matrix N_0 corresponding to the eigenvalue n_m^*:

$$U_m^+ T N_0 = n_m^* U_m^+ T, \qquad m = 1, \ldots, 4. \tag{30}$$

At this point it is convenient to use the Dirac notation for vectors in space $\mathbb{C}^4$ and for scalar product of these vectors

$$|U\rangle = U, \qquad \langle W| = W^+ T, \qquad \langle W|U\rangle = W^+ T U.$$

The scalar product defined with the use of a metric operator T can result in both positive and negative norm of the vectors, i.e. it corresponds to an indefinite metric[1] in space $\mathbb{C}^4$. Indeed,

$$\langle U_m|U_m\rangle = (\boldsymbol{H}^*_{\tau m} \quad \boldsymbol{q} \times \boldsymbol{E}^*_m) T \begin{pmatrix} \boldsymbol{H}_{\tau m} \\ \boldsymbol{q} \times \boldsymbol{E}_m \end{pmatrix} = 2\mathrm{Re}\, \boldsymbol{q}(\boldsymbol{E}^*_m \times \boldsymbol{H}_m) = \frac{16\pi}{c} \boldsymbol{q}\boldsymbol{S}_m, \tag{31}$$

where $\boldsymbol{S}_m = c\mathrm{Re}\,(\boldsymbol{E}^*_m \times \boldsymbol{H}_m)/(8\pi)$ is the time averaged Poynting vector of the mth eigenwave. The vector $\boldsymbol{S}_m$ involved in (31) can have both a positive and a negative projection onto the $\boldsymbol{q}$-direction (usually two eigenwaves with different polarization have a positive projection of $\boldsymbol{S}$ and the other two have a negative one).

It follows from (27) and (30) that

$$(n_m - n_m^*)\langle U_m|U_m\rangle = 0, \qquad m = 1, \ldots, 4.$$

Consequently, the following two cases are possible: (i) $\langle U_m|U_m\rangle \neq 0$ and n_m is real, (ii) $\langle U_m|U_m\rangle = 0$ and n_m is complex. The former corresponds to propagation of usual bulk eigenwave in the effective layer. In the latter case the eigenwave is an inhomogeneous surface (evanescent) wave with the energy flux parallel to the interfaces ($\boldsymbol{q}\boldsymbol{S}_m = 0$). If such a wave is present then it is accompanied by another eigenwave with 'refractive index' n_m^*. We eliminate case (ii) from our further consideration, assuming that the angle of incidence φ does not exceed the critical angle for total reflection (if any), so all quantities n_m are real.

Thus, the eigenvectors $|U_m\rangle$ form a basis in the amplitude space $\mathbb{C}^4$ and can be positively normalized with the use of the adjoint vectors $\langle \overline{U}_m|$:

$$\langle U_j|U_m\rangle = \eta_j \delta_{jm}, \qquad \langle \overline{U}_j|U_m\rangle = \delta_{jm}, \qquad \langle \overline{U}_j| = \eta_j \langle U_j|, \qquad j, m = 1, \ldots, 4, \tag{32}$$

where $\eta_j = \mathrm{sgn}\langle U_j|U_j\rangle$ is the sign of the norm for the vector $|U_j\rangle$ ($\eta_j = \pm 1$), δ_{jm} being the Kronecker delta. The completeness condition of the basis have the form

$$E = \sum_{m=1}^{4} |U_m\rangle\langle \overline{U}_m|, \tag{33}$$

and the matrix N_0 is represented as

$$N_0 = N_0 E = \sum_{m=1}^{4} n_m |U_m\rangle\langle \overline{U}_m|. \tag{34}$$

Taking into account the equations (33), (34) and an integral representation of the Heaviside function

$$\theta(z) = \frac{1}{2\pi \mathrm{i}} \int_{-\infty}^{\infty} \mathrm{d}k \frac{\mathrm{e}^{\mathrm{i}kz}}{k - \mathrm{i}0},$$

[1] The indefinite metric in Hilbert space can be used, for instance, in a quantization procedure of free electromagnetic field [74].

we obtain the formulae for the free propagator $P^{(0)}(z)$ (24)

$$P^{(0)}(z) = \sum_{m=1}^{4} \mathrm{e}^{\mathrm{i}k_m z} |U_m\rangle\langle\overline{U}_m| \tag{35}$$

and the Green's function $P_\mathrm{c}(z)$ (25)

$$P_\mathrm{c}(z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}k \sum_{m=1}^{4} \frac{-\mathrm{i}\,\mathrm{e}^{\mathrm{i}(k+k_m)z}}{k-\mathrm{i}0} |U_m\rangle\langle\overline{U}_m| = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}k\,\mathrm{e}^{\mathrm{i}kz} \sum_{m=1}^{4} \frac{-\mathrm{i}|U_m\rangle\langle\overline{U}_m|}{k-k_m-\mathrm{i}0}. \tag{36}$$

In (35) and (36), the projections $k_m = k_0 n_m$ of the eigenwave vectors onto the $\boldsymbol{q}$-direction have been introduced. The integrands in (36) have singularities located in the complex plane slightly above the real k-axis. Since all k_m are real, one can transform the integral as seen in (36) while still integrating over the real k.

After substitution of equations (35) and (36) into (26) we arrive at the following formula for the propagator in the nth order of perturbation method:

$$P^{(n)}(z', z_0) = \sum_{j=1}^{4}\sum_{s=1}^{4} P^{(n)}(j,s)\,\mathrm{e}^{\mathrm{i}k_s z'} |U_s\rangle\langle\overline{U}_j|\mathrm{e}^{-\mathrm{i}k_j z_0}, \qquad n \geqslant 0, \tag{37}$$

where

$$\begin{aligned}
&P^{(0)}(j,s) = \delta_{js},\\
&P^{(1)}(j,s) = 2\pi\,\mathrm{i}k_0 \langle\overline{U}_s|\mathcal{N}(k_s-k_j)|U_j\rangle,\\
&P^{(n)}(j,s) = 2\pi(\mathrm{i}k_0)^n \int_{-\infty}^{\infty}\mathrm{d}k^{(n-1)}\int_{-\infty}^{\infty}\mathrm{d}k^{(n-2)}\cdots\int_{-\infty}^{\infty}\mathrm{d}k''\int_{-\infty}^{\infty}\mathrm{d}k'\langle\overline{U}_s|\mathcal{N}(k_s-k^{(n-1)})\\
&\qquad\qquad \times P_\mathrm{c}(k^{(n-1)})\mathcal{N}(k^{(n-1)}-k^{(n-2)})\cdots P_\mathrm{c}(k'')\mathcal{N}(k''-k')P_\mathrm{c}(k')\mathcal{N}(k'-k_j)|U_j\rangle,\\
&\qquad n \geqslant 2.
\end{aligned} \tag{38}$$

In (38), we have introduced the Fourier image of the perturbation $\mathcal{N}(z)$[2]

$$\mathcal{N}(k) = \frac{1}{2\pi}\int_{z_0}^{z'} \mathrm{d}z\,\mathrm{e}^{-\mathrm{i}kz}\mathcal{N}(z) \tag{39}$$

and the Green's function in the wave number representation

$$P_\mathrm{c}(k) = \sum_{m=1}^{4} \frac{-\mathrm{i}|U_m\rangle\langle\overline{U}_m|}{k-k_m-\mathrm{i}0}. \tag{40}$$

The matrix element $P^{(n)}(j,s)$ in (37) corresponds to the nth order contribution of the sth eigenwave leaving the layer if the jth eigenwave enters the layer. This matrix element is graphically represented by a Feynman diagram in figure 2. When writing down the expression for $P^{(n)}(j,s)$ $(n \geqslant 0)$ with the use of the diagram, the external input and output lines with wave numbers k_j and k_s are associated with eigenvectors $|U_j\rangle$ and $\langle\overline{U}_s|$, respectively. The internal line of the virtual wave with an arbitrary wave number k is associated with an operator $P_\mathrm{c}(k)/(2\pi)$. Integration over such wave numbers is implied. The perturbation of the inhomogeneous layer (dashed line) which results in a change of wave number by k is described by an operator $2\pi\,\mathrm{i}k_0\mathcal{N}(k)$.

For the integration over variables $k', \ldots, k^{(n-1)}$ in (38) the following formula is convenient:

$$\int_{-\infty}^{\infty}\mathrm{d}k\frac{f(k)}{k-a-\mathrm{i}0} = \mathrm{V.P.}\int_{-\infty}^{\infty}\mathrm{d}k\frac{f(k)}{k-a} + \mathrm{i}\pi f(a), \tag{41}$$

[2] In (39), the limits of integration can be formally changed to $-\infty$ and ∞ if one assumes $\mathcal{N}(z) = 0$ at $z \leqslant z_0$ and $z \geqslant z'$.

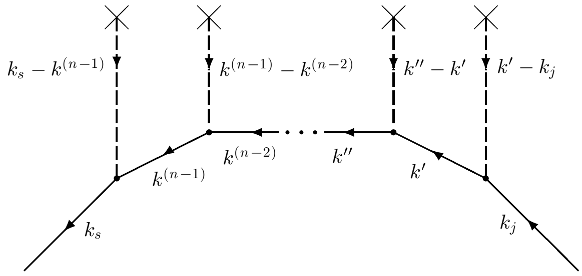

Figure 2. The Feynman diagram that corresponds to the n-order matrix element $P^{(n)}(j, s)$.

where sign V.P. on the right-hand side of (41) marks the Cauchy principal value of an improper integral.

Substituting the equations (22) and (37) into (12), we obtain the series expansion of the Fresnel's reflection and transmission operators

$$r = r^{(0)} + r^{(1)} + r^{(2)} + \cdots, \qquad d = d^{(0)} + d^{(1)} + d^{(2)} + \cdots.$$

As is obvious from (12), the pseudoinversion operation is used to calculate r and d. If a planar tensor q is expanded into a series $q = q^{(0)} + q^{(1)} + q^{(2)} + \cdots$ then a similar expansion of the pseudoinverse tensor $q^- = q^{-(0)} + q^{-(1)} + q^{-(2)} + \cdots$ can be calculated according to the formulae

$$\begin{aligned} q^{-(0)} &= (q^{(0)})^-, \qquad q^{-(1)} = -(q^{(0)})^- q^{(1)} (q^{(0)})^-, \\ q^{-(2)} &= (q^{(0)})^- [q^{(1)} (q^{(0)})^- q^{(1)} - q^{(2)}] (q^{(0)})^- \end{aligned} \tag{42}$$

and so on.

Finally, for the reflectance R (17) we have $R = R^{(0)} + R^{(1)} + R^{(2)} + \cdots$, where

$$\begin{aligned} R^{(0)} &= \mathrm{tr}(w r^{(0)} \Phi^{\mathrm{i}} r^{(0)+}), \qquad R^{(1)} = 2\mathrm{Re}\ \mathrm{tr}(w r^{(1)} \Phi^{\mathrm{i}} r^{(0)+}), \\ R^{(2)} &= \mathrm{tr}(w r^{(1)} \Phi^{\mathrm{i}} r^{(1)+}) + 2\mathrm{Re}\ \mathrm{tr}(w r^{(2)} \Phi^{\mathrm{i}} r^{(0)+}) \end{aligned} \tag{43}$$

and so on. Analogous formulae can be obtained for the transmittance D (19) too.

To demonstrate the technique proposed above we apply the general relations (22) and (37) to derive the reflectance and transmittance of a uniformly twisted uniaxial slab (section 4) as well as of one-dimensional inhomogeneous isotropic medium with arbitrary permittivity profile $\varepsilon(z)$ (section 5) for normal light incidence.

4. Reflectance and transmittance of uniformly twisted uniaxial slab in the first order of perturbation method

Optical properties of twisted crystals are described by the following dielectric permittivity tensor:

$$\varepsilon(z) = S(z) \varepsilon \widetilde{S}(z), \tag{44}$$

where

$$S(z) = \exp[\phi(z) \boldsymbol{d}^\times] = \boldsymbol{d} \otimes \boldsymbol{d} - \boldsymbol{d}^{\times 2} \cos\phi + \boldsymbol{d}^\times \sin\phi. \tag{45}$$

The operator $S(z)$ (45) rotates a vector by an angle of $\phi(z)$ around the axis directed along a unit vector $\boldsymbol{d}$. The tilde in (44) denotes transposition of the three-dimensional tensor.

An exact operator solution of wave equations for light that propagates along the axis of twisting in a crystal was obtained for the first time by Borzdov and Barkovsky [34]. Helicoidal bianisotropic media and their different realizations were proposed by Lakhtakia and Weiglhofer [75].

Here, we consider normal light incidence on a uniaxial twisted dielectric slab of thickness l with the axis of twisting perpendicular to the interfaces $z = z_0 = -l/2$ and $z = z' = l/2$ ($\boldsymbol{d} = \boldsymbol{q}$). We suppose that the optical axis of the uniaxial slab is parallel to the interfaces and the twisting is uniform so that $\phi(z) = az$ (a is a constant). If h is the period of a function $\varepsilon(z)$, i.e. $\varepsilon(z + h) = \varepsilon(z)$ then it is connected with a as $a = \pi/h$. The dielectric permittivity tensor ε involved in (44) has the form

$$\varepsilon = \varepsilon_\perp + (\varepsilon_\parallel - \varepsilon_\perp)\boldsymbol{b} \otimes \boldsymbol{b}, \tag{46}$$

Two principal values of ε are the same and equal to $\varepsilon_\perp$. The third principal value equals to $\varepsilon_\parallel$. After substitution of (46) into (44) and taking into account that $\boldsymbol{b}(z) \equiv S(z)\boldsymbol{b} = -\boldsymbol{q}^{\times 2}\boldsymbol{b}\cos\phi + \boldsymbol{q}^\times \boldsymbol{b}\sin\phi = \boldsymbol{b}\cos\phi - \boldsymbol{a}\sin\phi$, we obtain

$$\begin{aligned}\varepsilon(z) = \varepsilon_\perp + (\varepsilon_\parallel - \varepsilon_\perp)\boldsymbol{b}(z) \otimes \boldsymbol{b}(z) = \varepsilon_\perp + \tfrac{1}{2}\Delta\varepsilon[I + (\boldsymbol{b} \otimes \boldsymbol{b} - \boldsymbol{a} \otimes \boldsymbol{a})\cos 2az \\ - (\boldsymbol{b} \otimes \boldsymbol{a} + \boldsymbol{a} \otimes \boldsymbol{b})\sin 2az],\end{aligned}$$

where $\Delta\varepsilon = \varepsilon_\parallel - \varepsilon_\perp$ and $I = -\boldsymbol{q}^{\times 2} = \boldsymbol{b} \otimes \boldsymbol{b} + \boldsymbol{a} \otimes \boldsymbol{a}$. Propagation of electromagnetic waves in a twisted dielectric crystal is described by the general system of equations (2) where only two of the four elements (N_{12} and N_{21}) of the matrix $N(z)$ (3) will be nonzero for $\mu = 1, \alpha = \beta = 0$ and $\kappa = 0$ (normal incidence):

$$\begin{aligned}N_{12} &= -\boldsymbol{q}^\times \varepsilon(z)\boldsymbol{q}^\times = \varepsilon_\perp I + \tfrac{1}{2}\Delta\varepsilon[I - (\boldsymbol{b} \otimes \boldsymbol{b} - \boldsymbol{a} \otimes \boldsymbol{a})\cos 2az + (\boldsymbol{b} \otimes \boldsymbol{a} + \boldsymbol{a} \otimes \boldsymbol{b})\sin 2az], \\ N_{21} &= I, \qquad N_{11} = N_{22} = 0.\end{aligned} \tag{47}$$

Supposing that the slab under consideration is weakly anisotropic and ($\Delta\varepsilon \ll \varepsilon_\perp$) weakly inhomogeneous, we isolate the part of $N(z)$ that does not depend on z

$$N_0 = \begin{pmatrix} 0 & \varepsilon_\perp I \\ I & 0 \end{pmatrix}. \tag{48}$$

It describes propagation of electromagnetic waves in an effective homogeneous isotropic layer with dielectric permittivity $\varepsilon_\perp$. The coordinate-dependent part $\mathcal{N}(z)$ of the matrix $N(z)$ has zero elements $\mathcal{N}_{11}, \mathcal{N}_{21}$ and $\mathcal{N}_{22}$ and a nonzero element $\mathcal{N}_{12}$,

$$\mathcal{N}_{12}(z) = \tfrac{1}{2}\Delta\varepsilon[I - (\boldsymbol{b} \otimes \boldsymbol{b} - \boldsymbol{a} \otimes \boldsymbol{a})\cos 2az + (\boldsymbol{b} \otimes \boldsymbol{a} + \boldsymbol{a} \otimes \boldsymbol{b})\sin 2az]. \tag{49}$$

We see that the quantity $\Delta\varepsilon = \varepsilon_\parallel - \varepsilon_\perp$ plays a role of a small perturbation parameter.

The normalized eigenvectors of matrix (48) are

$$|U_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} \boldsymbol{b}\sqrt{n} \\ \boldsymbol{b}/\sqrt{n} \end{pmatrix}, \qquad |U_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} \boldsymbol{a}\sqrt{n} \\ \boldsymbol{a}/\sqrt{n} \end{pmatrix},$$
$$|U_3\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} \boldsymbol{b}\sqrt{n} \\ -\boldsymbol{b}/\sqrt{n} \end{pmatrix}, \qquad |U_4\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} \boldsymbol{a}\sqrt{n} \\ -\boldsymbol{a}/\sqrt{n} \end{pmatrix},$$

where $n = \sqrt{\varepsilon_\perp}$ is the refractive index of the effective isotropic layer. The eigenvalues corresponding to these eigenvectors are $n_1 = n_2 = n$ and $n_3 = n_4 = -n$. The vectors $|U_1\rangle$ and $|U_2\rangle$ describe the eigenwaves of the isotropic layer with mutually perpendicular $\boldsymbol{b}$- and $\boldsymbol{a}$-polarizations traveling in the positive direction of the z-axis while the vectors $|U_3\rangle$ and $|U_4\rangle$ describe the eigenwaves that travel in the negative direction.

It is not difficult to see that the norms $\langle U_j|U_j\rangle = 1$ for $j = 1, 2$ and $\langle U_j|U_j\rangle = -1$ for $j = 3, 4$ ($\eta_1 = \eta_2 = -\eta_3 = -\eta_4 = 1$). So, according to the third of the equations (32), we have

$$\langle\overline{U}_1| = \tfrac{1}{\sqrt{2}}(\boldsymbol{b}/\sqrt{n} \quad \boldsymbol{b}\sqrt{n}), \qquad \langle\overline{U}_2| = \tfrac{1}{\sqrt{2}}(\boldsymbol{a}/\sqrt{n} \quad \boldsymbol{a}\sqrt{n}),$$
$$\langle\overline{U}_3| = \tfrac{1}{\sqrt{2}}(\boldsymbol{b}/\sqrt{n} \quad -\boldsymbol{b}\sqrt{n}), \qquad \langle\overline{U}_4| = \tfrac{1}{\sqrt{2}}(\boldsymbol{a}/\sqrt{n} \quad -\boldsymbol{a}\sqrt{n}).$$

Owing to coincidence of eigenvalues n_1 and n_2 we can introduce the eigen subspace $\mathbb{C}^4_+$ of the amplitude space $\mathbb{C}^4$, spanned by vectors $|U_1\rangle$ and $|U_2\rangle$ and determined by the projective operator

$$Q_+ = |U_1\rangle\langle\overline{U}_1| + |U_2\rangle\langle\overline{U}_2| = \frac{1}{2}\begin{pmatrix} I & nI \\ I/n & I \end{pmatrix}. \tag{50}$$

The other eigen subspace $\mathbb{C}^4_-$ is spanned by vectors $|U_3\rangle$ and $|U_4\rangle$ and determined by the projective operator

$$Q_- = |U_3\rangle\langle\overline{U}_3| + |U_4\rangle\langle\overline{U}_4| = \frac{1}{2}\begin{pmatrix} I & -nI \\ -I/n & I \end{pmatrix}. \tag{51}$$

The operators Q_+ (50) and Q_- (51) satisfy the relations

$$Q_+^2 = Q_+, \qquad Q_-^2 = Q_-, \qquad Q_+Q_- = Q_-Q_+ = 0,$$
$$Q_+ + Q_- = E = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}, \qquad n(Q_+ - Q_-) = N_0.$$

Taking into account these relations along with (50), (51) and the first of the equations (38), the layer propagator in the zero-order approximation has the form (see (37))

$$P^{(0)} = \mathrm{e}^{\mathrm{i}kl}Q_+ + \mathrm{e}^{-\mathrm{i}kl}Q_- = \begin{pmatrix} I\cos kl & I\,\mathrm{i}n\sin kl \\ I\frac{\mathrm{i}}{n}\sin kl & I\cos kl \end{pmatrix}, \tag{52}$$

where $k = k_0 n = k_0\sqrt{\varepsilon_\perp}$. In the first order of perturbation method we have

$$P^{(1)} = 2\pi \mathrm{i}k_0[\mathrm{e}^{\mathrm{i}kl}Q_+\mathcal{N}(0)Q_+ + \mathrm{e}^{-\mathrm{i}kl}Q_-\mathcal{N}(0)Q_- + Q_+\mathcal{N}(2k)Q_- + Q_-\mathcal{N}(-2k)Q_+], \tag{53}$$

$\mathcal{N}(k)$ being the Fourier image of the matrix $\mathcal{N}(z)$. Applying formulae (39) with respect to (49), we establish that the only nonzero element of the matrix $\mathcal{N}(k)$ is

$$\mathcal{N}_{12}(k) = \frac{l\Delta\varepsilon}{4\pi}\left[f_k I - \frac{1}{2}(f_{k+2a} + f_{k-2a})(\boldsymbol{b}\otimes\boldsymbol{b} - \boldsymbol{a}\otimes\boldsymbol{a}) + \frac{\mathrm{i}}{2}(f_{k+2a} - f_{k-2a})(\boldsymbol{b}\otimes\boldsymbol{a} + \boldsymbol{a}\otimes\boldsymbol{b})\right],$$

where f_k is a function of the wave number k introduced as

$$f_k = \frac{\sin kl/2}{kl/2}, \qquad f_{k\pm 2a} = \frac{\sin(k\pm 2a)l/2}{(k\pm 2a)l/2}. \tag{54}$$

After a fairly simple calculation we find the elements of the matrix $P^{(1)}$ (53):

$$\begin{aligned}
P^{(1)}_{11} &= \frac{k_0 l\Delta\varepsilon}{8n}[-2\sin kl\,I + 2f_{2a}\sin kl(\boldsymbol{b}\otimes\boldsymbol{b} - \boldsymbol{a}\otimes\boldsymbol{a}) + (f_{2(k+a)} - f_{2(k-a)})(\boldsymbol{b}\otimes\boldsymbol{a} + \boldsymbol{a}\otimes\boldsymbol{b})],\\
P^{(1)}_{22} &= \frac{k_0 l\Delta\varepsilon}{8n}[-2\sin kl\,I + 2f_{2a}\sin kl(\boldsymbol{b}\otimes\boldsymbol{b} - \boldsymbol{a}\otimes\boldsymbol{a}) - (f_{2(k+a)} - f_{2(k-a)})(\boldsymbol{b}\otimes\boldsymbol{a} + \boldsymbol{a}\otimes\boldsymbol{b})],\\
P^{(1)}_{12} &= \mathrm{i}\frac{k_0 l\Delta\varepsilon}{8}[2(\cos kl + f_{2k})I - (2f_{2a}\cos kl + f_{2(k+a)} + f_{2(k-a)})(\boldsymbol{b}\otimes\boldsymbol{b} - \boldsymbol{a}\otimes\boldsymbol{a})],\\
P^{(1)}_{21} &= \mathrm{i}\frac{k_0 l\Delta\varepsilon}{8n^2}[2(\cos kl - f_{2k})I - (2f_{2a}\cos kl - f_{2(k+a)} - f_{2(k-a)})(\boldsymbol{b}\otimes\boldsymbol{b} - \boldsymbol{a}\otimes\boldsymbol{a})].
\end{aligned} \tag{55}$$

Now we calculate the reflection and transmission operators in the zero and first order of perturbation method. Let the crystal slab be surrounded by a homogeneous isotropic non-magnetic medium with dielectric permittivity ε'. For normal incidence of light, the surface impedance tensor (15) takes the form $\gamma' = I/n'$, where $n' = \sqrt{\varepsilon'}$ is the refractive index of the surrounding medium. The factors in formulae (12) for operators r and d will be expressed in terms of elements of P in the following way:

$$\begin{aligned} q &\equiv (I \quad -\gamma^{-})P\begin{pmatrix} I \\ -\gamma \end{pmatrix} = P_{11} + P_{22} - \frac{P_{12}}{n'} - n'P_{21}, \\ s &\equiv (I \quad \gamma^{-})P\begin{pmatrix} I \\ -\gamma \end{pmatrix} = P_{11} - P_{22} - \frac{P_{12}}{n'} + n'P_{21}. \end{aligned} \tag{56}$$

Substitution of the elements of $P^{(0)}$ (52) into (56) gives

$$q^{(0)} = I\left[2\cos kl - \mathrm{i}\left(n_{\mathrm{r}} + n_{\mathrm{r}}^{-1}\right)\sin kl\right], \qquad s^{(0)} = -I\,\mathrm{i}\left(n_{\mathrm{r}} - n_{\mathrm{r}}^{-1}\right)\sin kl,$$

where $n_{\mathrm{r}} = n/n'$ is the relative refraction index. Taking into account the first of formulae (42), we arrive at the reflection and transmission operators in the zero-order approximation:

$$r^{(0)} = -\frac{\mathrm{i}\left(n_{\mathrm{r}} - n_{\mathrm{r}}^{-1}\right)\sin kl}{2\cos kl - \mathrm{i}\left(n_{\mathrm{r}} + n_{\mathrm{r}}^{-1}\right)\sin kl}I, \qquad d^{(0)} = \frac{2}{2\cos kl - \mathrm{i}\left(n_{\mathrm{r}} + n_{\mathrm{r}}^{-1}\right)\sin kl}I. \tag{57}$$

The first-order correction $d^{(1)}$ to the transmission operator can be obtained when the elements (55) of the matrix $P^{(1)}$ are substituted into equation (56) for q and the second of the equations (42) is taken into consideration:

$$\begin{aligned} d^{(1)} = 2q^{-(1)} = \frac{k_0 l\Delta\varepsilon}{4n}&\left[2\cos kl - \mathrm{i}\left(n_{\mathrm{r}} + n_{\mathrm{r}}^{-1}\right)\sin kl\right]^{-2}\left\{\left[4\sin kl + 2\mathrm{i}\left(n_{\mathrm{r}} + n_{\mathrm{r}}^{-1}\right)\cos kl\right.\right. \\ &+ \left.2\mathrm{i}\left(n_{\mathrm{r}} - n_{\mathrm{r}}^{-1}\right)f_{2k}\right]I - \left[4\sin kl f_{2a} + 2\mathrm{i}\left(n_{\mathrm{r}} + n_{\mathrm{r}}^{-1}\right)\cos kl f_{2a}\right. \\ &+ \left.\left.\mathrm{i}\left(n_{\mathrm{r}} - n_{\mathrm{r}}^{-1}\right)(f_{2(k+a)} + f_{2(k-a)})\right](\boldsymbol{b}\otimes\boldsymbol{b} - \boldsymbol{a}\otimes\boldsymbol{a})\right\}. \end{aligned} \tag{58}$$

Calculation of the correction $r^{(1)}$ to the reflection operator is more complicated. For $s^{(1)}$ we have

$$\begin{aligned} s^{(1)} = \frac{k_0 l\Delta\varepsilon}{8n}&\left\{-2\mathrm{i}\left[\left(n_{\mathrm{r}} - n_{\mathrm{r}}^{-1}\right)\cos kl + \left(n_{\mathrm{r}} + n_{\mathrm{r}}^{-1}\right)f_{2k}\right]I + \mathrm{i}\left[2\left(n_{\mathrm{r}} - n_{\mathrm{r}}^{-1}\right)\cos kl f_{2a}\right.\right. \\ &+ \left.\left(n_{\mathrm{r}} + n_{\mathrm{r}}^{-1}\right)(f_{2(k+a)} + f_{2(k-a)})\right](\boldsymbol{b}\otimes\boldsymbol{b} - \boldsymbol{a}\otimes\boldsymbol{a}) \\ &+ \left.2(f_{2(k+a)} - f_{2(k-a)})(\boldsymbol{b}\otimes\boldsymbol{a} + \boldsymbol{a}\otimes\boldsymbol{b})\right\}. \end{aligned}$$

We expand the expression $r = sq^{-}$ and retain only the terms linear with respect to $\Delta\varepsilon$ in the first-order approximation:

$$\begin{aligned} r^{(1)} = s^{(1)}q^{-(0)} + s^{(0)}q^{-(1)} = \frac{k_0 l\Delta\varepsilon}{4n}&\left[2\cos kl - \mathrm{i}\left(n_{\mathrm{r}} + n_{\mathrm{r}}^{-1}\right)\sin kl\right]^{-2}\left\{-\left[4\sin kl f_{2k}\right.\right. \\ &+ \left.2\mathrm{i}\left(n_{\mathrm{r}} - n_{\mathrm{r}}^{-1}\right) + 2\mathrm{i}\left(n_{\mathrm{r}} + n_{\mathrm{r}}^{-1}\right)\cos kl f_{2k}\right]I + \left[2\sin kl(f_{2(k+a)} + f_{2(k-a)})\right. \\ &+ \left.2\mathrm{i}\left(n_{\mathrm{r}} - n_{\mathrm{r}}^{-1}\right)f_{2a} + \mathrm{i}\left(n_{\mathrm{r}} + n_{\mathrm{r}}^{-1}\right)(f_{2(k+a)} + f_{2(k-a)})\cos kl\right](\boldsymbol{b}\otimes\boldsymbol{b} - \boldsymbol{a}\otimes\boldsymbol{a}) \\ &+ \left.\left[2\cos kl - \mathrm{i}\left(n_{\mathrm{r}} + n_{\mathrm{r}}^{-1}\right)\sin kl\right](f_{2(k+a)} - f_{2(k-a)})(\boldsymbol{b}\otimes\boldsymbol{a} + \boldsymbol{a}\otimes\boldsymbol{b})\right\}. \end{aligned} \tag{59}$$

The general formulae (57)–(59) can be used to calculate the scalar reflectance and transmittance in the first-order approximation for arbitrary polarization of light incident on a slab. In particular, let us consider the nonpolarized light described by the beam tensor $\Phi^{\mathrm{i}} = -\frac{1}{2}\boldsymbol{q}^{\times 2} = \frac{1}{2}I$. When the light is normally incident then the tensor w involved in (17) and (19) coincides with I. Taking into account that the trace of the tensor I is equal to 2, we obtain in the zero-order approximation

$$
\begin{aligned}
R^{(0)} &= \frac{1}{2}\operatorname{tr}(r^{(0)}r^{(0)+}) = \frac{\left(n_\mathrm{r}-n_\mathrm{r}^{-1}\right)^2\sin^2 kl}{4+\left(n_\mathrm{r}-n_\mathrm{r}^{-1}\right)^2\sin^2 kl},\\
D^{(0)} &= \frac{1}{2}\operatorname{tr}(d^{(0)}d^{(0)+}) = \frac{4}{4+\left(n_\mathrm{r}-n_\mathrm{r}^{-1}\right)^2\sin^2 kl}.
\end{aligned}
\tag{60}
$$

The formulae (60) are well known and describe reflection and transmission of a homogeneous isotropic slab, and $R^{(0)} + D^{(0)} = 1$. Now using the relations in the form (43) along with formula (58) and taking into account that the trace of the tensor $\boldsymbol{b}\otimes\boldsymbol{b}-\boldsymbol{a}\otimes\boldsymbol{a}$ is zero, we find the correction to the transmittance,

$$
D_{\mathrm{NP}}^{(1)} = \operatorname{Re}\operatorname{tr}(d^{(1)}d^{(0)+}) = -\frac{2k_0 l\,\Delta\varepsilon\left(n_\mathrm{r}-n_\mathrm{r}^{-1}\right)\sin kl}{n\left[4+\left(n_\mathrm{r}-n_\mathrm{r}^{-1}\right)^2\sin^2 kl\right]^2}\left[\left(n_\mathrm{r}-n_\mathrm{r}^{-1}\right)\cos kl+\left(n_\mathrm{r}+n_\mathrm{r}^{-1}\right)\frac{\sin kl}{kl}\right],
\tag{61}
$$

where the subscript NP stands for nonpolarized radiation. The correction $R_{\mathrm{NP}}^{(1)}$ to the reflectance is calculated analogously with the use of equation (59) and turns out to be opposite in sign with respect to $D_{\mathrm{NP}}^{(1)}$: $R_{\mathrm{NP}}^{(1)} = -D_{\mathrm{NP}}^{(1)}$. It is noteworthy that the twist parameter a is not present in expressions for $R_{\mathrm{NP}}^{(1)}$ and $D_{\mathrm{NP}}^{(1)}$. For the normally incident *nonpolarized* light the slab inhomogeneity introduced by twisting does not affect the first-order corrections of perturbation method at all. This is not the case for *polarized* radiation. Let the radiation be linearly polarized, described by the beam tensor $\Phi^{\mathrm{i}} = \boldsymbol{h}\otimes\boldsymbol{h}$, where $\boldsymbol{h}$ is an unit real vector perpendicular to $\boldsymbol{q}$. Then the zero-order reflectance and transmittance coincide with those of (60), but in the first-order approximation

$$
\begin{aligned}
D_{\mathrm{L}}^{(1)} = -R_{\mathrm{L}}^{(1)} = D_{\mathrm{NP}}^{(1)} &+ \frac{k_0 l\,\Delta\varepsilon\left(n_\mathrm{r}-n_\mathrm{r}^{-1}\right)\sin kl}{n\left[4+\left(n_\mathrm{r}-n_\mathrm{r}^{-1}\right)^2\sin^2 kl\right]^2}\\
&\times\left[2\left(n_\mathrm{r}-n_\mathrm{r}^{-1}\right)f_{2a}\cos kl+\left(n_\mathrm{r}+n_\mathrm{r}^{-1}\right)\left(f_{2(k+a)}+f_{2(k-a)}\right)\right]\cos 2\theta,
\end{aligned}
\tag{62}
$$

where θ is the angle between the vectors $\boldsymbol{h}$ and $\boldsymbol{b}$. Here, the reflectance and transmittance corrections depend not only on a but also on the angle θ, which characterizes the polarization direction with respect to the helicoidal structure. Transition to the case of nonpolarized radiation corresponds to averaging of coefficients $D_{\mathrm{L}}^{(1)}$ and $R_{\mathrm{L}}^{(1)}$ over θ from 0 to 2π. As a result equation (62) transforms into (61).

Here, we do not calculate the reflectance and transmittance corrections $R^{(2)}$ and $D^{(2)}$ in the second order of perturbation method, which are quadratic with respect to $\Delta\varepsilon$. They become important when the dielectric permittivity ε' of the surrounding medium coincides with $\varepsilon_\perp$ involved in (46) ($n' = n, n_\mathrm{r} = 1$). For this case the corrections $R^{(1)}$ and $D^{(1)}$ linear with respect to $\Delta\varepsilon$ vanish. Note that according to the general formulae (37) and (38), the second-order correction to the propagator has the form

$$
\begin{aligned}
P^{(2)} = 2\pi(\mathrm{i}k_0)^2[&\mathrm{e}^{\mathrm{i}kl}Q_+\mathcal{N}^{(2)}(k,k)Q_+ + \mathrm{e}^{-\mathrm{i}kl}Q_-\mathcal{N}^{(2)}(-k,-k)Q_-\\
&+ Q_+\mathcal{N}^{(2)}(k,-k)Q_- + Q_-\mathcal{N}^{(2)}(-k,k)Q_+],
\end{aligned}
$$

where the following two-variable function is introduced:

$$
\mathcal{N}^{(2)}(k_{\mathrm{out}},k_{\mathrm{in}}) = \int_{-\infty}^{\infty}\mathrm{d}k'\mathcal{N}(k_{\mathrm{out}}-k')P_\mathrm{c}(k')\mathcal{N}(k'-k_{\mathrm{in}}),
\tag{63}
$$

and the Green's function in (63) is represented by the expression

$$P_{\mathrm{c}}(k') = -\mathrm{i}\left(\frac{Q_+}{k'-k-\mathrm{i}0} + \frac{Q_-}{k'+k-\mathrm{i}0}\right) = -\frac{\mathrm{i}}{(k'-\mathrm{i}0)^2 - k^2}\begin{pmatrix} k'I & nkI \\ kI/n & k'I \end{pmatrix}$$

(see also (40), (50) and (51)).

5. The first-order reflection and transmission of an inhomogeneous isotropic layer. Application to one-dimensional photonic crystals

Here, we derive the first-order perturbation corrections to the reflection and transmission factors of the electromagnetic wave normally incident on a weakly inhomogeneous layer with an arbitrary dielectric permittivity profile $\varepsilon(z)$. A rigorous solution of this problem can be obtained by considering of the equation

$$\frac{\mathrm{d}^2\boldsymbol{E}}{\mathrm{d}z^2} + \frac{\omega^2}{c^2}\varepsilon\boldsymbol{E} + \frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{1}{\varepsilon}\frac{\mathrm{d}\varepsilon}{\mathrm{d}z}\boldsymbol{E}\right) = 0 \tag{64}$$

and the corresponding boundary conditions (see, for example, [65]). As is known, equation (64) has closed-form solutions for only a few functions $\varepsilon(z)$. Approximate techniques for solving the wave equations of the type (64) are thus of importance [27, 65, 43], such as geometrical optics approximation, successive approximation technique, multiple reflection technique and some others.

Let a weakly inhomogeneous isotropic layer of thickness l be loss less (i.e., it is described by a real function $\varepsilon(z)$) and surrounded by a homogeneous isotropic medium with refractive index n'. Without loss of generality we consider $z = z_0 = -l/2$ and $z = z' = l/2$ as expressions for the plane interfaces. In the zero-order approximation the layer is substituted for an effective homogeneous isotropic medium with dielectric permittivity ε_0. As a result, the reflectance and transmittance in this approximation are given by formulae (60), where $n_{\mathrm{r}} = n/n'$, $k = k_0 n$ and $n = \sqrt{\varepsilon_0}$ is the refractive index of the effective medium. Generally speaking, the choice of the value for ε_0 is arbitrary but it is desirable for the difference $\varepsilon(z) - \varepsilon_0$ to be as small as possible for all $z \in (-l/2, l/2)$. We choose ε_0 as the layer-averaged dielectric permittivity,

$$\varepsilon_0 = \frac{1}{l}\int_{-l/2}^{l/2} \mathrm{d}z\, \varepsilon(z). \tag{65}$$

The constant part N_0 of the matrix $N(z)$ has the form (48), where $\varepsilon_\perp$ is replaced by ε_0. It is clear that the projective operators Q_+ and Q_- of the eigen-subspaces $\mathbb{C}^4_+$ and $\mathbb{C}^4_-$ are determined by formulae (50) and (51), respectively, where $n = \sqrt{\varepsilon_0}$ now.

Let us turn to calculation of the first-order approximation. The matrix $\mathcal{N}(z)$ has a single nonzero element $\mathcal{N}_{12}(z) = [\varepsilon(z) - \varepsilon_0]I$, and its Fourier image $\mathcal{N}(k)$ has a single nonzero element

$$\mathcal{N}_{12}(k) = \frac{\varepsilon_0 l}{2\pi}(\varepsilon_k - f_k)I. \tag{66}$$

In (66) the following notation:

$$\varepsilon_k = \frac{1}{\varepsilon_0 l}\int_{-l/2}^{l/2} \mathrm{d}z\, \mathrm{e}^{-\mathrm{i}kz}\varepsilon(z), \tag{67}$$

is introduced, f_k determined by the first equation in (54). Note that $\varepsilon_{-k} = \varepsilon_k^*$, since $\varepsilon(z)$ is a real function.

The layer propagator $P^{(1)}$ in the first-order approximation can be found according to formula (53). In this formula the Fourier image $\mathcal{N}(0)$ is zero owing to the choice of dielectric

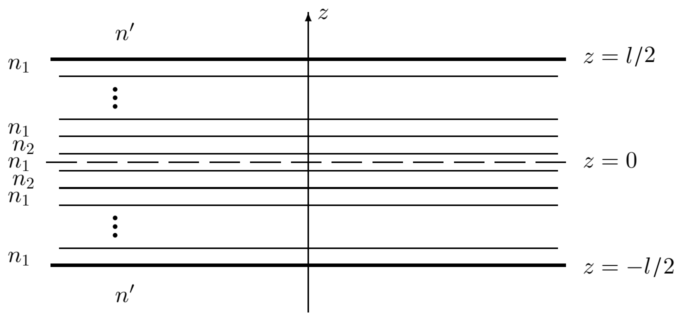

Figure 3. One-dimensional photonic crystal with refractive indices of layers n_1 and n_2.

permittivity ε_0 according to (65). The reflection and transmission operators as well as the scalar reflectance and transmittance are calculated in the same way as those for twisted crystals in the previous section. The final result is

$$R^{(1)} = -D^{(1)} = \frac{4kl\left(n_r^2 - n_r^{-2}\right)(\operatorname{Re}\varepsilon_{2k} - f_{2k})\sin kl}{\left[4 + \left(n_r - n_r^{-1}\right)^2 \sin^2 kl\right]^2}. \tag{68}$$

Thus, comparatively simple equations (68), (65) and (67) (see also (54)) determine the corrections to the reflection and transmission factors of arbitrary inhomogeneity profile $\varepsilon(z)$ in the first order of perturbation method. Formulae (68) are valid for arbitrary light polarization, since light incidence is normal.

Now let us consider a one-dimensional photonic crystal with N alternating layers of thickness $\Delta l = l/N$. Let the number of layers be $N = 4m + 1$, where $m = 0, 1, \ldots,$ and the function $\varepsilon(z)$ is even (see figure 3). We suppose that the refractive indices n_1 and n_2 of the layers differ slightly. According to formula (65) in the zero-order approximation such a crystal is substituted for an effective homogeneous isotropic medium with the refractive index

$$n = \sqrt{\frac{n_1^2 + n_2^2}{2} + \frac{n_1^2 - n_2^2}{2N}}. \tag{69}$$

In (67), the piecewise constant function $\varepsilon(z)$ will be integrated and such an integral is a sum of geometric progressions. As a result, in the case under consideration formula (68) for the reflectance and transmittance corrections takes the form

$$R^{(1)} = -D^{(1)} = \frac{2\left(n_r^2 - n_r^{-2}\right)\sin kl}{\left[4 + \left(n_r - n_r^{-1}\right)^2 \sin^2 kl\right]^2} \left(\frac{n_1^2 \sin \frac{N+1}{N}kl + n_2^2 \sin \frac{N-1}{N}kl}{n^2 \cos \frac{kl}{N}} - 2\sin kl \right). \tag{70}$$

Consequently, the reflection and transmission factors of the one-dimensional photonic crystal in the first order of perturbation method are equal to $R^{(0)} + R^{(1)}$ and $D^{(0)} + D^{(1)}$, respectively, where $R^{(0)}$ and $D^{(0)}$ are calculated by formulae (60).

6. Numerical comparison of approximate and exact solutions

Now we compare the approximate reflection factors for one-dimensional inhomogeneous twisted crystals (section 4) and photonic crystals (section 5) with the exact ones.

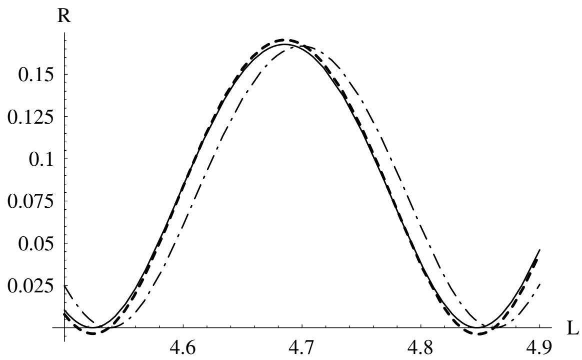

Figure 4. Dependence of the reflectance on the slab thickness $L = l/\lambda_0$ (parameter of twisting $A = 0.2$, polarization direction $\theta = 30^{\mathrm{o}}$). Solid line: the exact solution, dot-and-dash line: the zero order, dashed line: the first order.

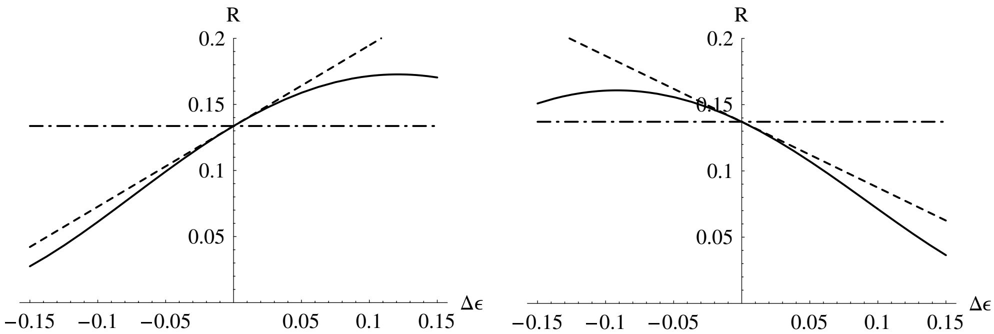

Figure 5. Dependence of the reflectance on the anisotropy parameter $\Delta\varepsilon$ for $L = 4.0$ (left) and $L = 4.1$ (right); $\varepsilon_\perp = 2.3798$, $A = 0.2$, $\theta = 30^{\mathrm{o}}$. The lines are as in figure 4.

For normal incidence of light on a twisted uniaxial crystal the matrix $N(z)$ of the equation system (2) has the tensor elements in the form of (47) and can be factorized in the following way:

$$N(z) = \begin{pmatrix} S(z) & 0 \\ 0 & S(z) \end{pmatrix} \begin{pmatrix} 0 & \sigma \\ I & 0 \end{pmatrix} \begin{pmatrix} \widetilde{S}(z) & 0 \\ 0 & \widetilde{S}(z) \end{pmatrix}, \tag{71}$$

where $\sigma = -\boldsymbol{q}^\times \varepsilon \boldsymbol{q}^\times = \varepsilon_\perp \boldsymbol{b} \otimes \boldsymbol{b} + \varepsilon_\parallel \boldsymbol{a} \otimes \boldsymbol{a}$ (see (46)) and $S(z) = \widetilde{S}(-z)$ is the rotation operator for the angle $\phi = az$ around the $\boldsymbol{q}$-axis. Such a representation of the matrix $N(z)$ makes it easy to find the layer propagator $P(z, -l/2)$ involved in the matrix solution of equation system (6),

$$P(z, -l/2) = \begin{pmatrix} S(z) & 0 \\ 0 & S(z) \end{pmatrix} \exp[\mathrm{i}k_0(z + l/2)M] \begin{pmatrix} S(l/2) & 0 \\ 0 & S(l/2) \end{pmatrix}. \tag{72}$$

The matrix M in (72) does not depend on z and has the form

$$M = \begin{pmatrix} \mathrm{i}A\boldsymbol{q}^\times & \sigma \\ I & \mathrm{i}A\boldsymbol{q}^\times \end{pmatrix}, \tag{73}$$

where a parameter $A = a/k_0$ is introduced, which represents the number of revolutions of the helicoidal structure per wavelength. Using the equation $\mathrm{d}S/\mathrm{d}z = aS\boldsymbol{q}^\times$, it is not difficult to make sure that the propagator $P(z, -l/2)$ is in accordance with equation (7). By setting $z = l/2$ in (72) we obtain the matrix $P = P(l/2, -l/2)$ involved in (12) for the reflection and transmission operators r and d. Then after substitution of the operator r into formula (17) at $w = I$, we find the reflectance R. The transmittance D is calculated as $D = 1 - R$, since energy losses in the crystal under consideration are absent.

Thus, the matrix P is expressed in terms of an exponent $\exp(\mathrm{i}k_0 l M)$, which can be calculated in closed form, for instance, with the use of spectral decomposition of the matrix M (73). We do not give here the somewhat intricate expressions for the elements of P. Instead, we make use of computer algebra systems to calculate numerically the matrix exponent $\exp(\mathrm{i}k_0 l M)$ for specified $\varepsilon_\perp$, $\varepsilon_\parallel$ and A. We then plot the dependence of the exact coefficient R on the media parameters.

Below we compare, on the one hand, the reflection coefficients $R^{(0)}$ and $R^{(0)} + R^{(1)}_{\mathrm{L}}$ for linearly polarized light in the zero- and first-order approximation according to formulae (60) and (62) and, on the other hand, the exact values of R according to (72), (12) and (17). We consider normal incidence of radiation with wavelength $\lambda_0 = 632.8$ nm on a twisted quartz slab with the refractive indices for ordinary waves $n = n_{\mathrm{o}} = 1.5427$ and for extraordinary waves $n_{\mathrm{e}} = 1.5517$ (respectively, $\varepsilon_\perp = n_{\mathrm{o}}^2 = 2.3798$, $\varepsilon_\parallel = n_{\mathrm{e}}^2 = 2.4078$ and $\Delta\varepsilon = 0.0280$). The slab is in air ($n' = 1$). In figure 4, these reflection coefficients are plotted as a function of the slab thickness $L = k_0 l/(2\pi)$ in units λ_0. The curve of the zero-order approximation turns out to be shifted to the right of the curve that corresponds to an exact solution. The reflectance of the first-order approximation becomes different from the exact one by more than 15% in the maxima of R for values $L \gtrsim 13$. The first-order approximation turns out to be inapplicable near the minima ($L \approx m/(2n)$, $m = 1, 2, \ldots$) since it gives negative reflectances there (see figure 4).

The zero-order approximation does not depend on the perturbation parameter $\Delta\varepsilon$, and the first-order approximation depends on this parameter linearly when other parameters are fixed (figure 5). The straight line corresponding to the linear approximation with respect to $\Delta\varepsilon$ is tangent to the curve of exact solution at point $\Delta\varepsilon = 0$. It means that formulae (62) represent the terms of a power series expansion of functions $R(\Delta\varepsilon)$ and $D(\Delta\varepsilon)$ which are linear with respect to $\Delta\varepsilon$. The higher-order corrections of the perturbation method will correspond to subsequent terms of the series expansion.

In figure 6, the dependence of exact and approximate reflectance on the refractive index n' of the surrounding medium is shown when other parameters are fixed. As was noted in section 4, the first-order approximation for coefficient R vanishes for $n' = n$. It is seen from figure 6 that this approximation can be negative at $n' \gtrsim n$. So in that case the higher-order corrections to R should be used.

In figure 7, the plots of R as a function of the incident wave polarization direction with respect to the vector $\boldsymbol{b}$ are shown. The relative error of the zero-order approximation for indicated values of the parameter of twisting and the slab thickness can reach 17%. For the first-order approximation it does not exceed 4%.

Finally, in figure 8 the plots of exact and approximate coefficient R as a function of the parameter of twisting $A = a/k_0$ are presented. At $A \approx n$ (i.e., at $a \approx k = nk_0$) the reflectance reaches a maximum or a minimum depending on the polarization direction of incident light. Resonance-like dependence of R on A is explained by wave interference on the helicoidal structure (period $h = \pi/a$ coincides with half-wave length for the structure $\lambda/2 = \pi/k$) and is described by a factor $f_{2(k-a)} \cos 2\theta = \frac{\sin(k-a)l}{(k-a)l} \cos 2\theta$ in formula (62). Note that at $\theta = 45^{\mathrm{o}} + 90^{\mathrm{o}}m$, where m is an integer, the first-order correction $R^{(1)}_{\mathrm{L}}$ does not depend on a and coincides with the correction $R^{(1)}_{\mathrm{NP}}$ for nonpolarized light (see horizontal dashed line in

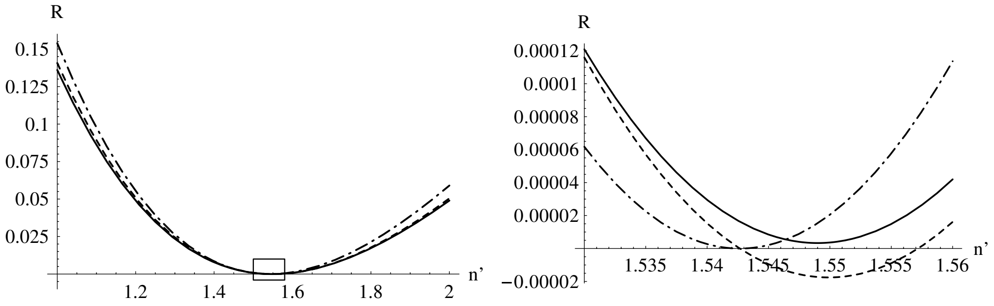

Figure 6. The reflectance of the quartz slab as a function of the refractive index n' of the surrounding medium (slab thickness $L = 7$, parameter of twisting $A = 0.2$, polarization direction $\theta = 30^o$). The right-hand part shows the enlarged plots near the minimum of R. The lines are as in figure 4.

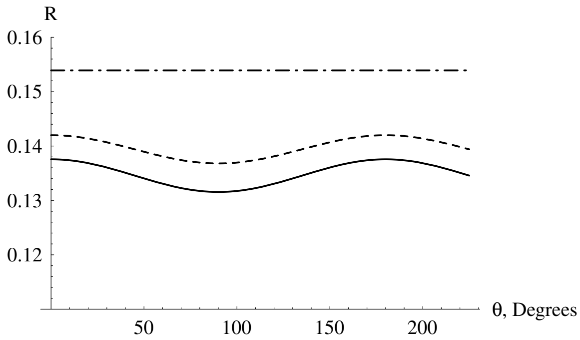

Figure 7. The reflectance of the slab for different light polarization directions (slab thickness $L = 7$, parameter of twisting $A = 0.2$, $n' = 1$). The lines are as in figure 4.

figure 8(c)). However, for these values of angle θ the resonance peak can be seen to remain in the exact solution. As calculation shows, in fact it stops to be strongly pronounced at $\theta = 50^o + 180^o m$ and $\theta = 130^o + 180^o m$ (solid line in figure 8(d)).

Thus, for the case of light propagation through a twisted uniaxial slab the first-order approximation of perturbation method sufficiently agrees with the exact reflectance at $L < 13$ and $|\Delta\varepsilon| < 0.08$ except when the reflectance is close to zero.

For one-dimensional photonic crystals the exact reflectance R and transmittance D are calculated by formulae (12), (17) and (19). In this connection the crystal propagator P can be found by numerical multiplication of propagators of the type (52) for separate layers (in formula (52) the refractive index n has to be replaced by the refractive indices n_1 or n_2 of the layers, and l by the thickness of the layer $\Delta l = l/N$). At the same time the zero- and first-order approximate values of R and D are calculated by formulae (60), (69) and (70). In figure 9, the dependence of exact and approximate reflectance of the photonic crystal on the normalized frequency $\Omega = k_0 \Delta l = \omega \Delta l / c$ of the normally incident electromagnetic wave in the vicinity of the first band gap is shown for different numbers of layers. It is supposed that the refractive indices of the layers n_1 and n_2 differ a little and almost do not depend on ω within this frequency band (small dispersion).

When the number of layers is not large ($N = 9$, see figure 9(a)), there is a weakly pronounced reflectance maximum at $\Omega \approx \pi/(2n)$, where n is the refractive index of the

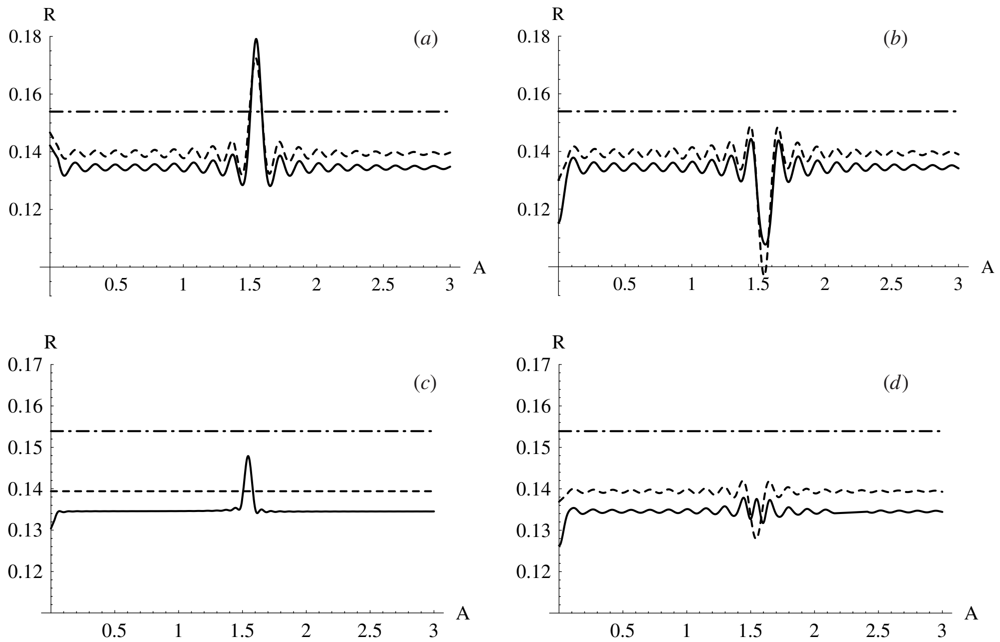

Figure 8. Dependence of the reflectance on the parameter of twisting A (slab thickness $L = 7, n' = 1$; polarization direction (*a*) $\theta = 30^{\mathrm{o}}$, (*b*) $\theta = 65^{\mathrm{o}}$, (*c*) $\theta = 45^{\mathrm{o}}$ and (*d*) $\theta = 50^{\mathrm{o}}$). The lines are as in figure 4.

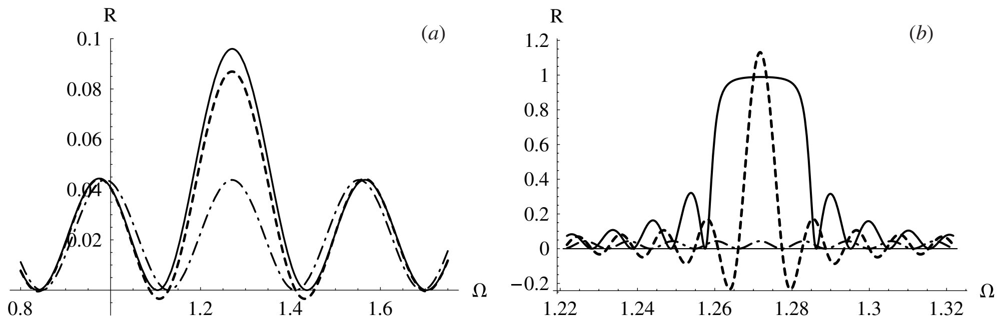

Figure 9. The reflectance of a one-dimensional photonic crystal as a function of the normalized wave frequency Ω (refractive indices $n_1 = 1.25, n_2 = 1.22$ and $n' = 1$). Number of layers is (*a*) $N = 9$ and (*b*) $N = 225$. The lines are as in figure 4.

effective isotropic medium. For this case the first-order approximation of perturbation method reasonably describes the dependence $R = R(\Omega)$. But if number of layers is rather large ($N = 225$, figure 9(*b*)), a whole band gap appears where the transmission of the photonic crystal is small. As seen in figure 9(*b*), the first-order approximation is poorly suited to describe the dependence $R = R(\Omega)$ within and around a band gap, although it provides the correct location for the band gap itself. Indeed, according to formula (70) the correction $R^{(1)}$ reaches its maximum when $\cos(kl/N) \approx 0$ or $\cos n\Omega \approx 0$. Therefore, the location of the mth band gap is $\Omega_m \approx \left(m + \frac{1}{2}\right)\frac{\pi}{n}, m \geqslant 1$. It is evident that at least the second-order approximation of perturbation method is needed to evaluate the forbidden bandwidth of various weakly inhomogeneous one-dimensional photonic crystals including those with defects. That is a subject of a separate investigation.

7. Conclusion

With the use of formulae (22), (37), (38) and (43) obtained above one can calculate in closed form the reflection and transmission of a wide variety of weakly inhomogeneous stratified linear bianisotropic media in an arbitrary order of perturbation method. These media can be either continuously inhomogeneous or multilayered with abrupt plane interfaces, but the optical properties of layers should not differ considerably. The technique proposed here is of importance as it is impossible to obtain an exact analytical solution of wave equations for the weakly inhomogeneous media in many cases.

We do not present a rigorous mathematical study of errors for our approximate solutions. These errors are determined by many factors. The most important of them is the degree of inhomogeneity of the type $\lambda\varepsilon^{-1}\Delta\varepsilon/\Delta z$, $\lambda\mu^{-1}\Delta\mu/\Delta z$ and so on, where λ is the wavelength. Note that this error can be different even for the same medium since it depends on how the constant matrix N_0 (21) of the zero-order approximation is chosen. For instance, for the twisted uniaxial crystals considered in section 4 the effective homogeneous isotropic medium characterized by the matrix N_0 in the form (48) was used as the zero-order approximation. Another choice of the matrix N_0 is also possible

$$N_0 = \begin{pmatrix} 0 & \sigma \\ I & 0 \end{pmatrix} = \begin{pmatrix} 0 & \varepsilon_\perp \boldsymbol{b}\otimes\boldsymbol{b} + \varepsilon_\parallel \boldsymbol{a}\otimes\boldsymbol{a} \\ I & 0 \end{pmatrix}$$

(see (71)) that corresponds to a non-twisted uniaxial crystal. It is clear that in this case the formulae for the reflection and transmission coefficients in the zero and higher orders of perturbation method will be more accurate, although the calculation of corrections will be more complicated.

For finite-thickness bianisotropic layers the corrections of perturbation method will be finite in any order as seen from formula (23) for the nth term of series for the propagator P.

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